Published by Institute of Physics Publishing for SISSA

Received: *June 28, 2007*
Revised: *September 7, 2007*
Accepted: *September 11, 2007*
Published: *September 19, 2007*

CP asymmetries with longitudinal and transverse beam polarizations in neutralino production and decay into the Z^0 boson at the ILC

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Abstract: We study neutralino production at the linear collider with the subsequent two-body decays $\tilde{\chi}_i^0 \to \tilde{\chi}_n^0 Z^0$ and $Z^0 \to \ell\bar{\ell}$, with $\ell = e, \mu, \tau$, or $Z^0 \to q\bar{q}$ with $q = c, b$. We show that transverse electron and positron beam polarizations allow the definition of unique CP observables. These are azimuthal asymmetries in the distributions of the final leptons or quarks. We calculate these CP asymmetries and the cross sections in the Minimal Supersymmetric Standard Model with complex higgsino and gaugino parameters μ and M_1. For final quark pairs, we find CP asymmetries as large as 30%. We discuss the significances for observing the CP asymmetries at the International Linear Collider (ILC). Finally we compare the CP asymmetries with those asymmetries which require unpolarized and/or longitudinally polarized beams only.

Keywords: Supersymmetry Phenomenology, CP violation, Supersymmetric Standard Model.

Contents

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1. Introduction

Supersymmetric (SUSY) models predict new particles with masses of the order of a few hundred GeV [1, 2]. Their discovery is a major goal of present and future colliders in the TeV range. In particular, the International e^+e^- Linear Collider (ILC) [3 − 7], with a center-of-mass energy of $\sqrt{s} = 500\,\mathrm{GeV}$ and an integrated luminosity of $\mathcal{L} = 500\,\mathrm{fb}^{-1}$ in the first stage, will precisely measure the masses and couplings of the SUSY particles when they are kinematically accessible. It has been shown that the underlying parameters of the SUSY model can be determined at the percent level and better [3, 7 − 9]. In particular, the option of polarized e^+ and e^- beams [10] at the ILC can yield higher statistics to test models beyond the Standard Model (SM). Transversely polarized beams allow to study additional observables which are sensitive to effects of new physics. These are, for example, models with extra spacial dimensions, specific triple-gauge boson couplings, and also new sources of CP violation [10].

In the Minimal Supersymmetric Standard Model (MSSM) [1, 2], the spin-half superpartners of the neutral gauge and CP-even Higgs bosons mix and form the four neutralinos

$\tilde{\chi}_i^0$. At tree-level, the neutralino sector of the MSSM is defined by the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ gaugino mass parameters M_1 and M_2, respectively, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields. Besides the sleptons, the superpartners of the leptons, and the charginos, the superpartners of the charged gauge and Higgs bosons, the neutralinos are expected to be among the lightest SUSY particles in many models. The neutralinos will be pair-produced [11 – 13] at the ILC

$$e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 , \quad i,j = 1, \ldots, 4 . \tag{1.1}$$

By measurements of the neutralino masses, cross sections and decay distributions, methods have been developed to determine the parameters of the neutralino sector [14 – 19]. If CP is violated, the parameters $M_1 = |M_1|\, e^{i\phi_{M_1}}$ and $\mu = |\mu|\, e^{i\phi_\mu}$ can be complex, while M_2 and $\tan\beta$ can then be chosen real and positive.

In general large values for CP phases in SUSY models lead to theoretical predictions for the electric dipole moments (EDM) of electron, neutron and that of the atoms ^{199}Hg and ^{205}Tl, which are close or beyond the current experimental upper bounds [20 – 22]. The restrictions on the phases from EDM measurements, however, strongly depend on the SUSY model, see e.g. [23, 24], and on the scenario [25, 26]. This means that SUSY CP phases of the order one are not ruled out by the present EDM experiments. For an unambiguous determination of the CP phases independent measurements are necessary, e.g., by analyzing CP sensitive observables at colliders, in particular at the ILC.

CP asymmetries with triple products [27, 28] have been analyzed for the production of neutralinos and for their various two-body [29 – 34] and three-body decays [18, 35 – 37]. It has been pointed out that longitudinally polarized beams can enhance simultaneously the cross sections and the CP asymmetries, such that higher statistics allows to measure even small phases [10].

In contrast to longitudinally polarized beams, the possibility of transverse beam polarization allows us to define a whole class of new and unique CP sensitive observables. These are asymmetries in the *azimuthal* distributions of final state particles. In general the transverse polarization of both beams is required, since these asymmetries vanish if only one beam is polarized. CP observables for transversely polarized beams have been studied for the production of selectrons [38] and charginos [39, 40]. For neutralino production, two-body decays have been analyzed: $\tilde{\chi}_j^0 \to \tilde{\ell}^{\pm}_{L,R}\ell^{\mp} \to \ell^+\ell^-\tilde{\chi}_1^0$ $(\ell = e, \mu)$ [34, 41], $\tilde{\chi}_j^0 \to \tilde{\chi}_1^0 h^0$ and $\tilde{\chi}_j^0 \to \tilde{\chi}_1^0 Z^0$ [41].

In this work we study CP asymmetries with triple products in neutralino production (1.1), followed by the decay of one of the neutralinos into the Z^0 boson

$$\tilde{\chi}_j^0 \to \tilde{\chi}_n^0 Z^0 , \quad n = 1 \quad \text{and} \quad j = 2, 3 , \tag{1.2}$$

for longitudinal and transverse beam polarizations. Due to the Majorana properties of the neutralinos, the angular distribution of the Z^0 boson is independent of the $\tilde{\chi}_j^0$ polarization, if the Z^0 polarization is summed [43, 29, 33, 42, 41]. Thus for longitudinally polarized beams, all CP sensitive information is lost. On the other hand, the CP sensitive observables

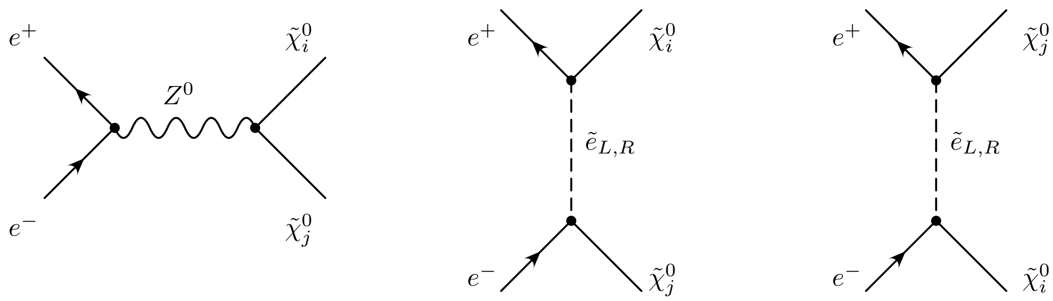

Figure 1: Feynman diagrams for neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ [11].

in the reaction (1.1) with transversely polarized beams require the reconstruction of the neutralino production plane. This can be challenging, since there is not enough kinematical information from energy and momentum conservation in the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 Z^0$ [34, 41]. Hence in either way, the polarization of the Z^0 has to be included and analyzed by the angular distributions of its decay products. On account of this, we study its leptonic and hadronic decays

$$Z^0 \to f\bar{f}\,, \quad f = \ell, q\,, \quad \ell = e, \mu, \tau\,, \quad q = c, b\,. \tag{1.3}$$

We will show that spin correlations between production and decay lead to sizeable CP asymmetries, which do not require the reconstruction of the production plane.

In section 2, we give the Lagrangians and couplings for neutralino production (1.1) and decay (1.2), (1.3). In section 3, we present the analytical formulae for the amplitude squared with polarized beams. We define the CP asymmetries for transversely and longitudinally polarized beams in section 4. In section 5, we present numerical results for the CP asymmetries and the cross sections. We compare the CP asymmetries with longitudinal and transverse beam polarizations, and show that transverse beam polarizations can help to determine the phases in the neutralino sector. In section 6, we give a summary and conclusions.

2. Lagrangians and couplings

In the MSSM, neutralino production (1.1) proceeds via Z^0 boson exchange in the s-channel, and selectron $\tilde{e}_{L,R}$ exchange in the t- and u-channels, see the Feynman diagrams in figure 1.

The Lagrangians for neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and decay $\tilde{\chi}_j^0 \to \tilde{\chi}_n^0 Z^0$ are [11, 12]

$$\mathcal{L}_{Z^0\tilde{\chi}_i^0\tilde{\chi}_j^0} = \frac{1}{2}\frac{g}{\cos\theta_W} Z_\mu^0 \bar{\tilde{\chi}}_i^0 \gamma^\mu [O_{ij}^{''L} P_L + O_{ij}^{''R} P_R]\tilde{\chi}_j^0\,, \quad i,j = 1,\ldots,4\,, \tag{2.1}$$

$$\mathcal{L}_{e\tilde{e}\tilde{\chi}_i^0} = g f_{ei}^L \bar{e} P_R \tilde{\chi}_i^0 \tilde{e}_L + g f_{ei}^R \bar{e} P_L \tilde{\chi}_i^0 \tilde{e}_R + \text{h.c.}\,, \tag{2.2}$$

$$\mathcal{L}_{Z^0 f\bar{f}} = -\frac{g}{\cos\theta_W} Z_\mu^0 \bar{f}\gamma^\mu [L_f P_L + R_f P_R] f\,, \quad f = \ell, q\,, \tag{2.3}$$

with $P_{L,R} = (1 \mp \gamma_5)/2$. In the photino, zino, Higgsino basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b)$, the couplings are

$$
\begin{aligned}
O''^{L}_{ij} &= -\frac{1}{2}\left[(N_{i3}N^*_{j3} - N_{i4}N^*_{j4})\cos 2\beta + (N_{i3}N^*_{j4} + N_{i4}N^*_{j3})\sin 2\beta\right] , && (2.4)\\
O''^{R}_{ij} &= -O''^{L*}_{ij} , && (2.5)\\
f^L_{ei} &= \sqrt{2}\left[\frac{1}{\cos\theta_W}\left(\frac{1}{2} - \sin^2\theta_W\right)N_{i2} + \sin\theta_W N_{i1}\right] , && (2.6)\\
f^R_{ei} &= \sqrt{2}\sin\theta_W\left[\tan\theta_W N^*_{i2} - N^*_{i1}\right] , && (2.7)\\
L_f &= T_{3f} - q_f\sin^2\theta_W , \quad R_f = -q_f\sin^2\theta_W , && (2.8)
\end{aligned}
$$

with the weak mixing angle θ_W, the weak coupling constant $g = e/\sin\theta_W$, $e > 0$, the electric charge q_f and isospin T_{3f} of fermion f, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two Higgs fields. The neutralino couplings $O''^{L,R}_{ij}$ and $f^{L,R}_{ei}$ contain the complex mixing elements N_{ij}, which diagonalize the neutralino matrix $N^*_{i\alpha}Y_{\alpha\beta}N^\dagger_{\beta k} = m_{\chi_i}\delta_{ik}$ [1], with the neutralino masses $m_{\chi_i} > 0$.

3. Distributions and cross section

The CP sensitive observables for transversal and longitudinal beam polarizations are asymmetries in the angular distributions of final state particles. They depend on the spin correlations between neutralino production and decay (1.1)–(1.3). The polarizations of the particles can be included using the spin density matrix formalism [44]. In this formalism, the amplitude squared for neutralino production has been calculated at tree level for unpolarized and longitudinally polarized beams [12], and for transversely polarized beams [34]. The subsequent decay of one of the neutralinos into a Z^0 boson, $\tilde{\chi}^0_j \to \tilde{\chi}^0_n Z^0$, followed by $Z^0 \to f\bar{f}$, has also been calculated in the density matrix formalism [43, 33, 29]. For completeness, we shortly summarize the results in the following.

The amplitude squared for neutralino production (1.1) and decay (1.2)–(1.3) is given by

$$
|T|^2 = 4\,|\Delta(\tilde{\chi}^0_j)|^2\,|\Delta(Z^0)|^2\left[P\ D + \sum_{b=1}^{3}\Sigma^b_P\ \Sigma^b_D\right] , \tag{3.1}
$$

with the propagators of the decaying neutralino $\tilde{\chi}^0_j$ and Z^0 boson

$$
\Delta(\tilde{\chi}^0_j) = \frac{i}{s_{\chi_j} - m^2_{\chi_j} + i m_{\chi_j}\Gamma_{\chi_j}} , \quad \Delta(Z^0) = \frac{i}{s_Z - m^2_Z + i m_Z\Gamma_Z} , \tag{3.2}
$$

where Γ_Z and Γ_{χ_j} are the total decay widths of the associated particles and s_Z and s_{χ_j} are their invariant masses. The differential cross section then reads

$$
d\sigma = \frac{1}{2s}|T|^2 d\mathrm{Lips}(s; p_{\chi_j}, p_{\chi_n}, p_f, p_{\bar{f}}) , \tag{3.3}
$$

where dLips is the Lorentz invariant phase-space element, given in appendix B.

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The amplitude squared (3.1) has contributions from neutralino production (P) and decay (D). The function P is independent of the polarizations of the neutralinos, whereas Σ_P^b depends on the polarization of neutralino $\tilde{\chi}_j^0$. The longitudinal polarization of the neutralino is given by Σ_P^3/P, the transverse polarization of the neutralino in the production plane is given by Σ_P^1/P and the polarization perpendicular to the production plane is given by Σ_P^2/P. It is convenient to decompose the functions P and Σ_P^b into unpolarized (unpol), longitudinally (L) and transversely polarized (T) contributions of the electron and positron beams:

$$P = P_{\rm unpol} + P_L + P_T\,, \quad \Sigma_P^b = \Sigma_{P,\rm unpol}^b + \Sigma_{P,L}^b + \Sigma_{P,T}^b\,. \tag{3.4}$$

The quantities $P_{\rm unpol}$, P_L and $\Sigma_{P,\rm unpol}^b$, $\Sigma_{P,L}^b$ are given in [12], and the quantities P_T and $\Sigma_{P,T}^b$ have been calculated in [34].

The quantities $\Sigma_{P,T}^b$ with transversal beam polarization depend on the polarization of the neutralino $\tilde{\chi}_j^0$ and are[1]

$$\Sigma_{P,T}^b = \Sigma_P^b(Z\tilde{e}_L)_T + \Sigma_P^b(Z\tilde{e}_R)_T + \Sigma_P^b(\tilde{e}_L\tilde{e}_R)_T\,, \tag{3.5}$$

where

$$\begin{aligned}
\Sigma_P^b(Z\tilde{e}_L)_T &= \mathcal{P}_T^-\mathcal{P}_T^+\ \frac{1}{2}\frac{g^4}{\cos^4\theta_W}R_e \\
&\quad\times\{{\rm Re}(\Delta(Z)f_{ei}^{L*}f_{ej}^{L}O_{ij}^{\prime\prime L}[\Delta(\tilde{e}_L,u)^* - \Delta(\tilde{e}_L,t)^*])\ r_1^b \\
&\quad-\ {\rm Im}(\Delta(Z)f_{ei}^{L*}f_{ej}^{L}O_{ij}^{\prime\prime L}[\Delta(\tilde{e}_L,u)^* + \Delta(\tilde{e}_L,t)^*])\ r_2^b\}\,, \qquad (3.6)\\
\Sigma_P^b(Z\tilde{e}_R)_T &= \mathcal{P}_T^-\mathcal{P}_T^+\ \frac{1}{2}\frac{g^4}{\cos^4\theta_W}L_e \\
&\quad\times\{{\rm Re}(\Delta(Z)f_{ei}^{R*}f_{ej}^{R}O_{ij}^{\prime\prime R}[\Delta(\tilde{e}_R,u)^* - \Delta(\tilde{e}_R,t)^*])\ r_1^b \\
&\quad+\ {\rm Im}(\Delta(Z)f_{ei}^{R*}f_{ej}^{R}O_{ij}^{\prime\prime R}[\Delta(\tilde{e}_R,u)^* + \Delta(\tilde{e}_R,t)^*])\ r_2^b\}\,, \qquad (3.7)\\
\Sigma_P^b(\tilde{e}_L\tilde{e}_R)_T &= \mathcal{P}_T^-\mathcal{P}_T^+\ \frac{1}{4}g^4 \\
&\quad\times\{{\rm Re}([\Delta(\tilde{e}_L,t)\Delta(\tilde{e}_R,u)^* - \Delta(\tilde{e}_L,u)\Delta(\tilde{e}_R,t)^*]f_{ei}^{L*}f_{ej}^{L}f_{ei}^{R*}f_{ej}^{R})\ r_1^b \\
&\quad+{\rm Im}([\Delta(\tilde{e}_L,t)\Delta(\tilde{e}_R,u)^* + \Delta(\tilde{e}_L,u)\Delta(\tilde{e}_R,t)^*]f_{ei}^{L*}f_{ej}^{L}f_{ei}^{R*}f_{ej}^{R})\ r_2^b\}\,, \qquad (3.8)
\end{aligned}$$

with $-1 \le \mathcal{P}_T^\pm \le 1$ $[(\mathcal{P}_T^\pm)^2 + (\mathcal{P}_L^\pm)^2 \le 1]$ being the degree of transverse $e^\pm$ beam polarizations. The selectron propagators are $\Delta(\tilde{e}_{L,R},t) = i/(t - m_{\tilde{e}_{L,R}}^2)$ and $\Delta(\tilde{e}_{L,R},u) = i/(u - m_{\tilde{e}_{L,R}}^2)$, with the Mandelstam variables $t = (p_{\chi_j} - p_{e^-})^2$ and $u = (p_{\chi_i} - p_{e^-})^2$. For a center-of mass energy $\sqrt{s}$ far beyond the Z^0-threshold, the Z^0-width can be neglected in the propagator $\Delta(Z^0) = i/(s - m_Z^2)$. The kinematical factors in eqs. (3.6)–(3.8) are

$$\begin{aligned}
r_1^b &= m_{\chi_j}\{[(t_-\cdot s^b)(t_+\cdot p_{\chi_i}) + (t_-\cdot p_{\chi_i})(t_+\cdot s^b)](p_{e^-}\cdot p_{e^+}) \\
&\quad+[(p_{e^-}\cdot s^b)(p_{e^+}\cdot p_{\chi_i}) + (p_{e^-}\cdot p_{\chi_i})(p_{e^+}\cdot s^b) - (p_{e^-}\cdot p_{e^+})(p_{\chi_i}\cdot s^b)](t_-\cdot t_+)\}\,, \qquad (3.9)\\
r_2^b &= \varepsilon_{\mu\nu\rho\sigma}\ m_{\chi_j}[t_+^\mu p_{e^-}^\nu p_{e^+}^\rho s^{b,\sigma}(t_-\cdot p_{\chi_i}) + t_-^\mu p_{e^-}^\nu p_{e^+}^\rho p_{\chi_i}^\sigma(t_+\cdot s^b) \\
&\quad+t_-^\mu t_+^\nu p_{e^+}^\rho s^{b,\sigma}(p_{e^-}\cdot p_{\chi_i}) + t_-^\mu t_+^\nu p_{e^-}^\rho p_{\chi_i}^\sigma(p_{e^+}\cdot s^b)]\,, \qquad (3.10)
\end{aligned}$$

[1]These terms are also given in [34], however, there in the terms $\Sigma_P^b(Z\tilde{e}_{L,R})_T$ and $P(Z\tilde{e}_{L,R})_T$ a factor $g^2/\cos^2\theta_W$ is missing.

where $\varepsilon_{0123} = -1$. The $e^{\pm}$ polarization vectors $t_{\pm}$ are given in eq. (A.2), and the neutralino polarization vectors are given in eq. (A.5), for details see appendix A.

In [34], the properties of $\Sigma^b_{P,T}$ (3.5) have been discussed in detail. In contrast to unpolarized and longitudinally polarized beams, it contains no contributions from pure selectron exchange, $\Sigma^b_P(\tilde{e}_L\tilde{e}_L)_T$ and $\Sigma^b_P(\tilde{e}_R\tilde{e}_R)_T$. However, transverse beam polarization leads to $\tilde{e}_L$-$\tilde{e}_R$ interference $\Sigma^b_P(\tilde{e}_L\tilde{e}_R)_T$, which is absent for unpolarized and longitudinally polarized beams [12]. In addition, there is no contribution $\Sigma^b_P(ZZ)_T$ owing to the Majorana character of the neutralinos. Note that in the high energy limit $m_e/\sqrt{s} \to 0$, the neutralino polarization is proportional to the product of transverse beam polarizations $\Sigma^b_{P,T} \propto \mathcal{P}^-_T\mathcal{P}^+_T$.

The contributions to the amplitude squared, eq. (3.1), from the neutralino decay are

$$D = \frac{4g^4}{\cos^4\theta_W}(L_f^2+R_f^2)\Big\{2|O''^L_{nj}|^2(p_{\chi_j}\cdot p_{\bar{f}})[m^2_{\chi_j}-m^2_{\chi_i}+m^2_Z-2(p_{\chi_j}\cdot p_{\bar{f}})] \\ -|O''^L_{nj}|^2\frac{m_Z^2}{2}(m^2_{\chi_j}-m^2_{\chi_i}+m^2_Z)+[\mathrm{Re}(O''^L_{nj})^2-\mathrm{Im}(O''^L_{nj})^2]m_{\chi_i}m_{\chi_j}m_Z^2\Big\}, \quad (3.11)$$

and

$$\Sigma^b_D = \frac{4g^4}{\cos^4\theta_W}(L_f^2-R_f^2)\Big\{|O''^L_{nj}|^2 m_{\chi_j}\Big[(m^2_{\chi_j}-m^2_{\chi_i}-m^2_Z)(p_{\bar{f}}\cdot s^b_{\chi_j})-(p_Z\cdot s^b_{\chi_j})(p_{\chi_j}\cdot p_{\bar{f}}) \\ +m_Z^2(p_Z\cdot s^b_{\chi_j})\Big]-4\mathrm{Im}(O''^L_{nj})\mathrm{Re}(O''^L_{nj})m_{\chi_i}\varepsilon_{\mu\nu\rho\sigma}s^{b,\mu}_{\chi_j}p^\nu_Z p^\rho_{\chi_j}p^\sigma_{\bar{f}} \\ -[\mathrm{Re}(O''^L_{nj})^2-\mathrm{Im}(O''^L_{nj})^2]m_{\chi_i}\Big[(m^2_{\chi_j}-m^2_{\chi_i}+m^2_Z)(p_{\bar{f}}\cdot s^b_{\chi_j})-(p_Z\cdot s^b_{\chi_j})(p_{\chi_j}\cdot p_{\bar{f}})\Big]\Big\}. \quad (3.12)$$

Different ways to calculate the squared amplitude $|T|^2$ for the decay chain $\tilde{\chi}^0_j \to \tilde{\chi}^0_n Z^0$, followed by $Z^0 \to f\bar{f}$, with complete spin correlations in the spin density matrix formalism, have been given in detail in [43, 33, 29]. In contrast to previous calculations, we directly obtain the terms D and Σ^b_D for the entire neutralino decay chain by summing over the internal polarization vectors ϵ^μ_Z of the propagating Z^0 boson in the narrow width approximation.

4. CP asymmetries

For the combined process of neutralino production and decay (1.1)–(1.3), the spin correlations $\Sigma^b_P\,\Sigma^b_D$ in the amplitude squared $|T|^2$ (3.1), allow us to define several CP sensitive observables. These are asymmetries in the azimuthal angular distribution of the final state fermions from the Z^0 boson decay (1.3). The CP asymmetries are based on different triple product correlations, which we classify in the following according to the beam polarization. We will define CP asymmetries for transversely as well as for longitudinally and/or unpolarized beams.

4.1 Transverse beam polarizations

We define a CP observable for transverse beam polarizations which is based on the T-odd correlation

$$\mathcal{O}_T = \vec{t}_+\cdot(\hat{p}_f\times\hat{p}_{e^-})(\vec{t}_-\cdot\hat{p}_f)+\vec{t}_-\cdot(\hat{p}_f\times\hat{p}_{e^-})(\vec{t}_+\cdot\hat{p}_f) = \sin(\eta-2\phi_f), \quad (4.1)$$

where $\hat{p}_{e^-}$ is the unit vector of the e^- beam, $\hat{p}_f$ the unit vector of the final fermion f in $Z^0 \to f\bar{f}$, ϕ_f is the azimuthal angle of f, and the constant $\eta = \phi_+ + \phi_-$ is the sum of the two azimuthal angles of the polarization vectors of the positron beam, $\vec{t}_+$, and the electron beam, $\vec{t}_-$. The equality $\mathcal{O}_T = \sin(\eta - 2\phi_f)$ in eq. (4.1) follows from the specific parametrization of the momenta in the center-of-mass system, for their definition see appendix A. Note that the T-odd correlation $\mathcal{O}_T$ is contained in the kinematical factor r_2^b, eq. (3.10), entering in the neutralino polarization terms $\Sigma^b_{P,T}$, given in eqs. (3.6)–(3.8). The CP asymmetry is then defined by

$$\mathcal{A}_f^{\mathrm{T}} = \frac{N[\mathcal{O}_T > 0] - N[\mathcal{O}_T < 0]}{N[\mathcal{O}_T > 0] + N[\mathcal{O}_T < 0]}\,, \tag{4.2}$$

where the number of events with $\mathcal{O}_T > 0\,(<0)$ is denoted by $N[\mathcal{O}_T > 0\,(<0)\,]$. With the definition of the cross section, eq. (3.3), $\mathcal{L}\ \mathrm{d}\sigma = \mathrm{d}N$, where $\mathcal{L}$ is the integrated luminosity, and the amplitude squared (3.1) with the decompositions of the quantities P and Σ^b_P, eqs. (3.4) and (3.5), we obtain for the CP asymmetry

$$\mathcal{A}_f^{\mathrm{T}} = \frac{\int \mathrm{sgn}[\mathcal{O}_T]\ |T|^2\ d\mathrm{Lips}}{\int |T|^2\ d\mathrm{Lips}} = \frac{\int \mathrm{sgn}[\mathcal{O}_T]\ \Sigma^b_{P,T}\ \Sigma^b_D\ d\mathrm{Lips}'}{\int (P_{\mathrm{unpol}} + P_L)\ D\ d\mathrm{Lips}'}\,, \tag{4.3}$$

summed over $b = 1, 2, 3$, and with $d\mathrm{Lips}' = d\mathrm{Lips}/(ds_{\chi_j} ds_Z)$ (B.1), where we have already used the narrow width approximation for the propagators (B.5). In the numerator of $\mathcal{A}_f^{\mathrm{T}}$, eq. (4.3), the spin correlations $\Sigma^b_{P,T}\Sigma^b_D$ are only non-vanishing if weighted with $\mathrm{sgn}[\mathcal{O}_T]$, as they include $\mathcal{O}_T = \sin(\eta - 2\phi_f)$. In the denominator of eq. (4.3), we have used $\int P_T\ D\ d\mathrm{Lips}' = 0$ since the cross section is independent of the transverse beam polarizations. Also the terms of the amplitude squared which depend on the $\tilde{\chi}^0_j$ polarization vanish for an integration over the entire phase space. The asymmetry $\mathcal{A}_f^{\mathrm{T}}$ is sensitive to CP violating couplings in the production only, which enter $\Sigma^b_{P,T}$, see eq. (3.5). That means $\mathcal{A}_f^{\mathrm{T}} = 0$ for the production of a pair of equal neutralinos $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_i$. In addition, the asymmetry $\mathcal{A}_f^{\mathrm{T}}$ is proportional to the product of transverse beam polarizations

$$\mathcal{A}_f^{\mathrm{T}} \propto \Sigma^b_{P,T} \propto \mathcal{P}_T^- \mathcal{P}_T^+\,, \tag{4.4}$$

and thus vanishes if one beam is unpolarized.

For the measurement of $\mathcal{A}_f^{\mathrm{T}} = -\mathcal{A}_{\bar{f}}^{\mathrm{T}}$, the charges and the flavors of the final fermions f and $\bar{f}$ have to be distinguished. For $f = e, \mu$ this will be possible on an event by event basis. For $f = \tau$ it will be possible after taking into account corrections due to the reconstruction of the τ momentum. For $f = q$ the distinction of the quark flavors should be possible by flavor tagging of the heavy quarks $q = b, c$ [46, 48]. The distinction of the heavy quark charges can be accomplished with very good precision in the case of semi-leptonic decays of the hadrons. For the majority of b and c-jets, it also can be accomplished by the reconstruction of the vertex charge in the cases where the hadrons decay non-leptonically [48, 45, 49]. However, b and c tagging will be essential.

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The asymmetry for final quarks $\mathcal{A}_q^{\rm T}$ is always larger than that for final leptons $\mathcal{A}_\ell^{\rm T}$, due to the dependence of $\mathcal{A}_f^{\rm T}$ on the $Z^0 f\bar{f}$ couplings [43, 33, 29]

$$\mathcal{A}_f^{\rm T} \propto \frac{R_f^2 - L_f^2}{R_f^2 + L_f^2} \quad \Rightarrow \quad \mathcal{A}_{b(c)}^{\rm T} = \frac{R_\ell^2 + L_\ell^2}{R_\ell^2 - L_\ell^2} \frac{R_{b(c)}^2 - L_{b(c)}^2}{R_{b(c)}^2 + L_{b(c)}^2} \mathcal{A}_\ell^{\rm T} \tag{4.5}$$

$$\Rightarrow \quad \mathcal{A}_{b(c)}^{\rm T} \simeq 6.3 \; (4.5) \times \mathcal{A}_\ell^{\rm T} \,, \tag{4.6}$$

which follows from eqs. (3.11), (3.12) and (4.2).

4.2 Longitudinal or unpolarized beams

For longitudinal or unpolarized beams, the triple product [36]

$$\mathcal{O}_L = \vec{p}_{e^-} \cdot (\vec{p}_f \times \vec{p}_{\bar{f}}) \,, \tag{4.7}$$

can be used to define the CP asymmetry

$$\mathcal{A}_f^{\rm L} = \frac{\int \mathrm{sgn}[\mathcal{O}_L] \; |T|^2 \; d\mathrm{Lips}}{\int |T|^2 \; d\mathrm{Lips}} = \frac{\int \mathrm{sgn}[\mathcal{O}_L] \; (\Sigma_{P,\mathrm{unpol}}^b + \Sigma_{P,L}^b) \; \Sigma_D^b \; d\mathrm{Lips}'}{\int (P_{\mathrm{unpol}} + P_L) \; D \; d\mathrm{Lips}'} \,. \tag{4.8}$$

This CP asymmetry has been analyzed in detail in [33]. Due to the different angular dependence of the triple product $\mathcal{O}_L$, eq. (4.7), and the CP-odd terms in $\Sigma_{P,T}^b$, we have $\int \mathrm{sgn}[\mathcal{O}_L] \; \Sigma_{P,T}^b \; \Sigma_D^b \; d\mathrm{Lips}' = 0$. This means that the asymmetry $\mathcal{A}_f^{\rm L}$ does not depend on the degree of transverse beam polarization. In contrast to the asymmetry $\mathcal{A}_f^{\rm T}$, eq. (4.3), the asymmetry $\mathcal{A}_f^{\rm L}$ is also sensitive to the CP violating couplings in the neutralino decay $\tilde{\chi}_j^0 \to \tilde{\chi}_n^0 Z^0$ [33]. Due to the dependence of $\mathcal{A}_f^{\rm L}$ on the $Z^0 f\bar{f}$ couplings, we again have [43, 33, 29]

$$\mathcal{A}_{b(c)}^{\rm L} \simeq 6.3 \; (4.5) \times \mathcal{A}_\ell^{\rm L} \,. \tag{4.9}$$

4.3 Measurability of the CP asymmetries

For a measurement of non-vanishing phases in the neutralino sector at the ILC, not only large CP asymmetries but also large cross sections are required. Since the cross section appears in the denominator of the CP asymmetries $\mathcal{A}_f^{\rm T}$, eq. (4.3), and $\mathcal{A}_f^{\rm L}$, eq. (4.8), we are often confronted with a situation where large cross sections lead to small asymmetries and vice versa. In order to estimate whether the CP asymmetries can be measured at the ILC, we consider their statistical significances [43, 33, 29].

The significance for the CP asymmetry $\mathcal{A}_f^{\rm T}$ with transversely polarized beams is defined by

$$S_f^{\rm T} = |\mathcal{A}_f^{\rm T}| \sqrt{\mathcal{L} \; \sigma(e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{\mathrm{unpol}} \, \mathrm{BR}(\tilde{\chi}_j^0 \to \tilde{\chi}_n^0 Z^0) \, \mathrm{BR}(Z^0 \to f\bar{f})} \,. \tag{4.10}$$

Note that only the unpolarized cross section $\sigma(e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{\mathrm{unpol}}$ appears in eq. (4.10), since the cross section is independent of transversal degrees of beam polarizations. We define the statistical significances for the asymmetries $\mathcal{A}_f^{\rm L}$ by

$$S_f^{\rm L} = |\mathcal{A}_f^{\rm L}| \sqrt{\mathcal{L} \; \sigma(e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0) \, \mathrm{BR}(\tilde{\chi}_j^0 \to \tilde{\chi}_n^0 Z^0) \, \mathrm{BR}(Z^0 \to f\bar{f})} \,. \tag{4.11}$$

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Here, the cross section depends on the degree of longitudinal electron and positron beam polarizations. Note that S_f^{T} and S_f^{L}, in short S_f, are larger for $f = b, c$ than for $f = \ell = e, \mu, \tau$ with [43, 33, 29]

$$S_b \simeq 7.7 \times S_\ell \quad \text{and} \quad S_c \simeq 4.9 \times S_\ell\,, \tag{4.12}$$

caused by larger asymmetries (4.5), (4.10), and by larger branching ratios $\mathrm{BR}(Z^0 \to b\bar{b}) \simeq 1.5 \times \sum_\ell \mathrm{BR}(Z^0 \to \ell\bar{\ell})$, and $\mathrm{BR}(Z^0 \to c\bar{c}) \simeq 1.2 \times \sum_\ell \mathrm{BR}(Z^0 \to \ell\bar{\ell})$, $\ell = e, \mu, \tau$, with $\sum_\ell \mathrm{BR}(Z^0 \to \ell\bar{\ell}) \simeq 0.1$ [20].

For an ideal detector, a significance of, e.g., $S_f = 1$ implies that $\mathcal{A}_f$ can be measured at the statistical 68% confidence level. However, the definition of our theoretical significance is based on statistics only, and does not include detector and particle reconstruction efficiencies. Our significances are thus upper bounds to judge on the feasibility to measure the CP asymmetries. In order to predict the absolute values of confidence levels, detailed Monte Carlo analyses including detector and background simulations with particle identification and reconstruction efficiencies would be required. A Monte Carlo analysis for a CP asymmetry in the production and decay of neutralinos with longitudinal polarized beams has been carried out in [35]. However, such an analysis is beyond the scope of the present work. We only estimate how the detection rates for b- and c-quark jets have to be modified. Using vertex detectors, flavor tagging of b- and c-quarks is possible and the corresponding efficiencies and purities have been studied [45]. It has been shown [46], that b-quarks (c-quarks) can be identified with an efficiency of 50% (50%) at a purity of 90% (80%) for $e^+e^- \to q\bar{q}$ at $\sqrt{s} = 500\,\mathrm{GeV}$. This would lead to a reduction of our statistical significance S_b (S_c) by a factor of 0.64 (0.57).

5. Numerical results

We present numerical results for the CP asymmetry $\mathcal{A}_f^{\mathrm{T}}$, eq. (4.2), with transverse beam polarizations, and for the CP asymmetry $\mathcal{A}_f^{\mathrm{L}}$, eq. (4.8), with longitudinal beam polarizations. For neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_{2(3)}^0$ and decay $\tilde{\chi}_{2(3)}^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$, we study the dependences of $\mathcal{A}_b^{\mathrm{T}}$ and the cross section on the gaugino mass parameter $M_1 = |M_1|e^{i\phi_{M_1}}$ and on the higgsino mass parameter $\mu = |\mu|e^{i\phi_\mu}$, as well as on the beam polarizations $\mathcal{P}(e^-)$ and $\mathcal{P}(e^+)$. Throughout this study we take $\sqrt{s} = 500\,\mathrm{GeV}$. The values of the CP asymmetries for final leptons $\mathcal{A}_\ell^{\mathrm{T}}$, and those for final c-quarks $\mathcal{A}_c^{\mathrm{T}}$, are related to $\mathcal{A}_b^{\mathrm{T}}$ by eq. (4.5). Finally we compute cross sections and asymmetries which are accessible with longitudinally polarized beams, and compare them with the results for transversely polarized beams.

For the calculation of the neutralino widths and branching ratios, we include the two-body decays [29]

$$\tilde{\chi}_i^0 \to \tilde{\ell}_n \ell,\ \tilde{\nu}_\ell \nu_\ell,\ \tilde{\chi}_1^0 Z^0,\ \tilde{\chi}_1^\mp W^\pm,\ \tilde{\chi}_1^0 H_1^0\,, \tag{5.1}$$

with $n = R, L$ for $\ell = e, \mu$, and $n = 1, 2$ for $\ell = \tau$. The Higgs mass parameter is chosen $m_A = 1\,\mathrm{TeV}$. With such a high value for m_A explicit CP violation in the Higgs sector is

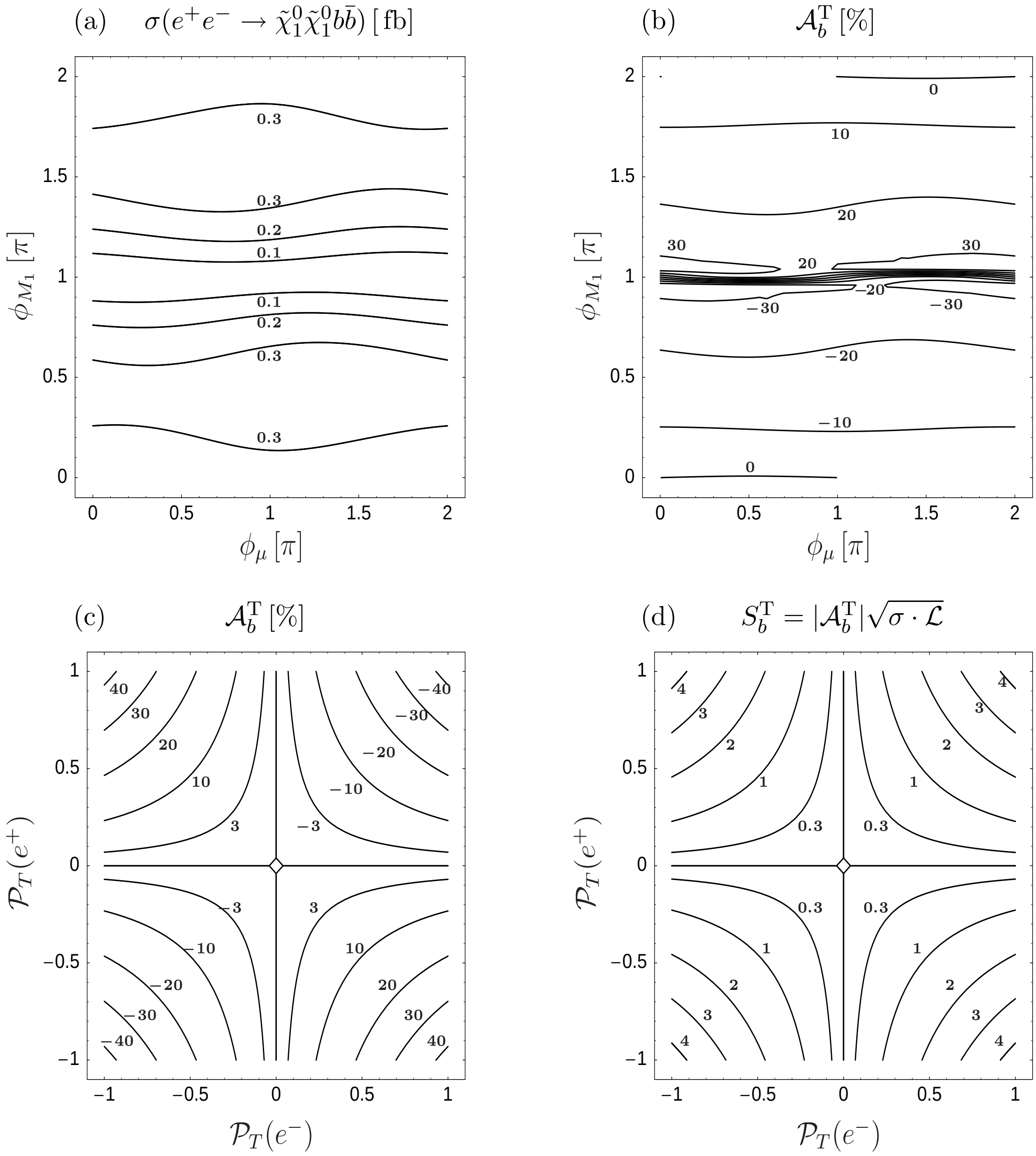

Figure 2: Neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$, at $\sqrt{s} = 500\,\mathrm{GeV}$ for Scenario A given in table 1. Phase dependence of (a) the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b}) \equiv \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0) \cdot \mathrm{BR}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0) \cdot \mathrm{BR}(Z^0 \to \bar{b}b)$, and (b) the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$ for transverse beam polarizations $\mathcal{P}_T(e^-) = 0.9$, $\mathcal{P}_T(e^+) = 0.6$. Contour lines in the $\mathcal{P}_T(e^-)$–$\mathcal{P}_T(e^+)$ plane for (c) the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$, and (d) the statistical significance S_b^{T}, with $\phi_{M_1} = \frac{3\pi}{4}$, $\phi_\mu = 0$, and an integrated luminosity $\mathcal{L} = 500\,\mathrm{fb}^{-1}$.

not important for the lightest Higgs state H_1^0 [47]. In the stau sector, we fix the trilinear scalar coupling parameter $A_\tau = 250\,\mathrm{GeV}$. To reduce the number of free parameters, we take $|M_1| = (5/3)\ M_2 \tan^2\theta_W$, inspired by gaugino mass unification.

5.1 Neutralino $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production and decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0 \to \tilde{\chi}_1^0 b\bar{b}$

In figure 2a, we show contour lines of the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b}) \equiv \sigma(e^+e^- \to$

Scenario	M_2	$\lvert\mu\rvert$	$\tan\beta$	$m_{\tilde{e}_L}$	$m_{\tilde{e}_R}$	$m_{\chi_1^0}$	$m_{\chi_2^0}$	$m_{\chi_3^0}$	$m_{\chi_1^\pm}$
A	380	120	50	250	210	107	119	205	114
B	250	200	7	240	220	119	163	213	161
C	280	340	3	350	250	141	236	349	234

Table 1: Input parameters M_2, $|\mu|$, $\tan\beta$, $m_{\tilde{e}_L}$, and $m_{\tilde{e}_R}$ for Scenarios A, B, and C, with $|M_1| = (5/3)\ M_2 \tan^2\theta_W$. The neutralino and chargino masses are calculated with $\phi_{M_1} = \frac{3\pi}{4}$ and $\phi_\mu = 0$. All mass parameters are given in GeV.

$\tilde{\chi}_1^0\tilde{\chi}_3^0) \cdot \mathrm{BR}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0) \cdot \mathrm{BR}(Z^0 \to \bar{b}b)$ in the ϕ_μ–ϕ_{M_1} plane for Scenario A, see table 1. The production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)_{\mathrm{unpol}}$ as well as the decay branching ratio $\mathrm{BR}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0)$ (which varies between 5% and 15%) strongly depend on ϕ_{M_1} but only mildly on ϕ_μ, and their minimum values are obtained for $\phi_{M_1} = \pi$ and $\phi_\mu = 0$. In figure 2b we show the phase dependence of the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$, eq. (4.2). The asymmetry $\mathcal{A}_b^{\mathrm{T}}$ reaches its maximum of about 36% at $\phi_{M_1} = 1.05\pi$ and $\phi_\mu = 1.8\pi$. The maximum of the asymmetry $\mathcal{A}_b^{\mathrm{T}}$ is close to $\phi_{M_1} = \pi$. The reason for this is that the cross section, which is the denominator of the asymmetry, has a minimum there. We remark that the CP asymmetry is asymmetric with respect to the transformations $\phi_{M_1} \to 2\pi - \phi_{M_1}$ and $\phi_\mu \to 2\pi - \phi_\mu$, while any CP-even observable, e.g. the cross section in figure 2a, must be symmetric under these transformations. Figure 2c shows the dependence of $\mathcal{A}_b^{\mathrm{T}} \propto \mathcal{P}_T(e^-) \cdot \mathcal{P}_T(e^+)$ on the degrees of transverse polarizations of the electron and positron beams for Scenario A with $\phi_{M_1} = \frac{3\pi}{4}$ and $\phi_\mu = 0$. The maximum value of the asymmetry $\mathcal{A}_b^{\mathrm{T}} \approx \pm 40\%$ is attained for the maximal degree of polarizations $\mathcal{P}_T(e^+) = \pm 1$ and $\mathcal{P}_T(e^-) = \mp 1$. In figure 2d we show the statistical significance S_b, eq. (4.10), for an integrated luminosity of $\mathcal{L} = 500\,\mathrm{fb}^{-1}$. A measurement of the asymmetry thus requires a high degree of the beam polarizations. In the following we assume that a polarization of 90% for the electron beam, and 60% for the positron beam is feasible, and study the $|\mu|$–M_2 dependence of the asymmetry $\mathcal{A}_b^{\mathrm{T}}$.

In figure 3a, we show contour lines of the production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$ in the $|\mu|$–M_2 plane, with $\phi_{M_1} = \frac{3\pi}{4}$ and $\phi_\mu = 0$. The other MSSM parameters are fixed as defined in Scenario A of table 1. In figure 3b we show the contour lines of the branching ratio $\mathrm{BR}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0)$. It is as large as 90% for $M_2 \sim |\mu| \sim 170\,\mathrm{GeV}$. For $|\mu| \gtrsim M_2$ and $|\mu| \gtrsim 200\,\mathrm{GeV}$ the decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^\pm W^\mp$ dominates. Figure 3c shows the corresponding CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$ in the $|\mu|$–M_2 plane. The asymmetry $\mathcal{A}_b^{\mathrm{T}}$ attains large values in the parameter region where the lighter neutralino states $\tilde{\chi}_{1,2}^0$ are higgsino-like. For instance, for $|\mu| \approx 140\,\mathrm{GeV}$ and $M_2 \approx 420\,\mathrm{GeV}$, the asymmetry is $\mathcal{A}_b^{\mathrm{T}} \approx -20\%$. In addition, we show in figure 3d the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$ $(\equiv \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0) \cdot \mathrm{BR}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0) \cdot \mathrm{BR}(Z^0 \to \bar{b}b))$. For $|\mu| \approx 150\,\mathrm{GeV}$ and $M_2 \approx 170\,\mathrm{GeV}$, we obtain the largest values up to 7 fb, where the asymmetry $\mathcal{A}_b^{\mathrm{T}}$ reaches 8.5%. With these values of $\mathcal{A}_b^{\mathrm{T}}$ and σ, the statistical significance, eq. (4.10), is $S_b^{\mathrm{T}} \approx 5$, for an integrated luminosity of $\mathcal{L} = 500\,\mathrm{fb}^{-1}$.

Now we discuss $\mathcal{A}_b^{\mathrm{T}}$ in neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to \bar{b}b$, for Scenario B, see table 1, with a small value for $\tan\beta = 7$. In this scenario, the neutralino $\tilde{\chi}_3^0$ is a strong mixture of gaugino and higgsino components, and $\mathrm{BR}(\tilde{\chi}_3^0 \to$

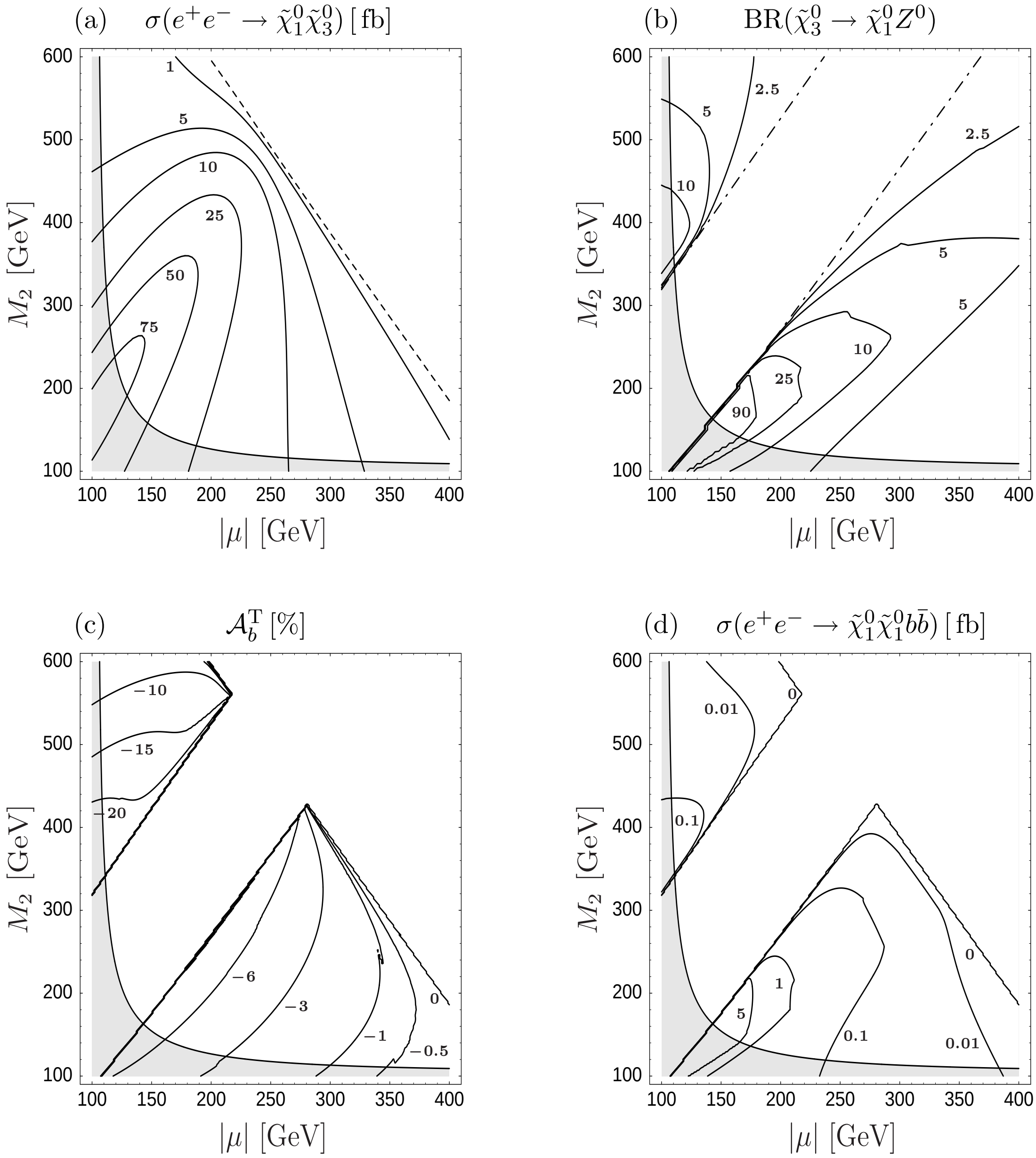

Figure 3: Contour lines in the $|\mu|$–M_2 plane of (a) the production cross section $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3)$, (b) the branching ratio $\mathrm{BR}(\tilde{\chi}^0_3 \to \tilde{\chi}^0_1 Z)$, (c) the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$, and (d) the total cross section $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_1 b\bar{b})$, for Scenario A given in table 1, with $\phi_{M_1} = \frac{3\pi}{4}$, $\phi_\mu = 0$, at $\sqrt{s} = 500\,\mathrm{GeV}$ with transverse beam polarizations $\mathcal{P}_T(e^-) = 0.9$, $\mathcal{P}_T(e^+) = 0.6$. In the gray shaded area $m_{\chi_1^\pm} <$ 104 GeV. The dashed line in (a) indicates the kinematical limit $m_{\chi^0_1} + m_{\chi^0_3} = \sqrt{s}$, and the dashed-dotted line in (b) indicates the limit $m_{\chi^0_3} = m_{\chi^0_1} + m_{Z^0}$.

$\tilde{\chi}^0_1 Z^0) = 100\%$. In figure 4a we show contour lines of the production cross section $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3)$ in the ϕ_μ–ϕ_{M_1} plane. The production cross section ranges from 25 fb, for $\phi_{M_1} = \pi$ and $\phi_\mu = 0$, to 54 fb, for $\phi_{M_1} = \phi_\mu = 0$. In figure 4b, we show the total cross section $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_1 b\bar{b})$, which varies between 4 fb and 8 fb. Contour lines for the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$ are shown in figure 4c. As can be seen, $\mathcal{A}_b^{\mathrm{T}}$ can attain large values even for CP phases close to the CP conserving case. For instance, for $\phi_\mu = 0$ and $\phi_{M_1} = 0.8\pi$, the asymmetry reaches about -10%. The corresponding statistical significance, shown in figure 4d, reaches $S_b^{\mathrm{T}} = 5$, for $\mathcal{L} = 500\,\mathrm{fb}^{-1}$.

5.1.1 Comparison of transversely and longitudinally polarized beams

In this subsection we compare the asymmetry $\mathcal{A}_b^{\mathrm{T}}$, eq. (4.2), that is obtained with transverse beam polarization with the asymmetry $\mathcal{A}_b^{\mathrm{L}}$, eq. (4.8), for longitudinally or unpolarized beams. Figure 5 shows the contours of the asymmetry $\mathcal{A}_b^{\mathrm{L}}$ and the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$ for longitudinally polarized beams in the $|\mu|$–M_2 plane for scenario A, see table 1. The asymmetry $\mathcal{A}_b^{\mathrm{L}}$ can reach $\pm 10\%$, while the total cross section can reach 15 fb. By comparing these results for longitudinal beam polarizations with the results of figure 3 for transverse beam polarizations, we find that in general for higgsino-like scenarios the statistical significance of asymmetry $\mathcal{A}_b^{\mathrm{T}}$ is larger than that for asymmetry $\mathcal{A}_b^{\mathrm{L}}$. The reason is that for higgsino-like scenarios the leading contribution to the asymmetry $\mathcal{A}_b^{\mathrm{T}}$ is due to the $\tilde{e}_L$–$\tilde{e}_R$ interference term, eq. (3.8), which is absent for longitudinally polarized beams. Clearly, for larger values of the selectron masses $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$, the significance of the asymmetry $\mathcal{A}_b^{\mathrm{T}}$ is reduced due to a suppression of the $\tilde{e}_L$–$\tilde{e}_R$ interference term, as these masses enter in the propagators, see figure 1. On the other hand, for gaugino-like scenarios, the statistical significance of asymmetry $\mathcal{A}_b^{\mathrm{L}}$, is larger than that for $\mathcal{A}_b^{\mathrm{T}}$, since then the $\tilde{e}_R$–$\tilde{e}_R$ and the $\tilde{e}_L$–$\tilde{e}_L$ terms, which are only present for unpolarized and longitudinally polarized beams, give the dominant contribution to $\mathcal{A}_b^{\mathrm{L}}$.

In figure 6, we show the phase dependence of the production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$, the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$, the asymmetry $\mathcal{A}_b^{\mathrm{L}}$ and the corresponding significance for longitudinally polarized beams $(\mathcal{P}_{e^-}^L, \mathcal{P}_{e^+}^L) = (+0.9, -0.6)$ in scenario A of table 1. The asymmetry $\mathcal{A}_b^{\mathrm{L}}$ can reach $\pm 15\%$, the total cross section can be as large as $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b}) = 16\,\mathrm{fb}$, such that the significance can reach $S_b^{\mathrm{L}} = 12$. In comparing these results, figure 6, with the results for transversely polarized beams, see figure 4, we find that the asymmetry $\mathcal{A}_b^{\mathrm{L}}$ can be measured with higher statistics compared to $\mathcal{A}_b^{\mathrm{T}}$. The reason is that the high degree of longitudinally polarized beams, $(\mathcal{P}_{e^-}^L, \mathcal{P}_{e^+}^L) = (0.9, -0.6)$, enhances both the production cross section and the asymmetry, whereas transversely polarized beams do not change the cross section, see figure 4.

5.2 Neutralino $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production and decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0 \to \tilde{\chi}_1^0 b\bar{b}$

In figure 7, we show the phase dependence of the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$, eq. (4.2), and the cross section for neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$, and decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$, for Scenario C, see table 1. The asymmetry reaches a maximum value of about 4.5% for $(\phi_{M_1}, \phi_\mu) = (-\frac{\pi}{4}, \frac{\pi}{4})$. The production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ varies from 19 fb for $(\phi_{M_1}, \phi_\mu) = (0,0)$, to 72 fb for $(\phi_{M_1}, \phi_\mu) = (\pi, \pi)$. The branching ratio is maximal $\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0) = 100\%$ for $\phi_\mu \lesssim 0.4\pi$ and $\phi_\mu \gtrsim 1.6\pi$. For $0.4\pi \gtrsim \phi_\mu \lesssim 1.6\pi$, the $\tilde{\chi}_2^0$ decay channels $\tilde{\chi}_2^0 \to \tilde{\ell}_R \ell$ are kinematical accessible, which leads to a strong ϕ_μ dependence of the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$ around the threshold region. In the region where $\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0) = 100\%$ ($\phi_\mu \lesssim 0.4\pi$ and $\phi_\mu \gtrsim 1.6\pi$) the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$ varies from about 4 fb to 10 fb, see figure 7b. For Scenario C, with phases $(\phi_{M_1}, \phi_\mu) = (-\frac{\pi}{4}, \frac{\pi}{4})$, the significance is $S_b \approx 2$, for $\mathcal{L} = 500\,\mathrm{fb}^{-1}$.

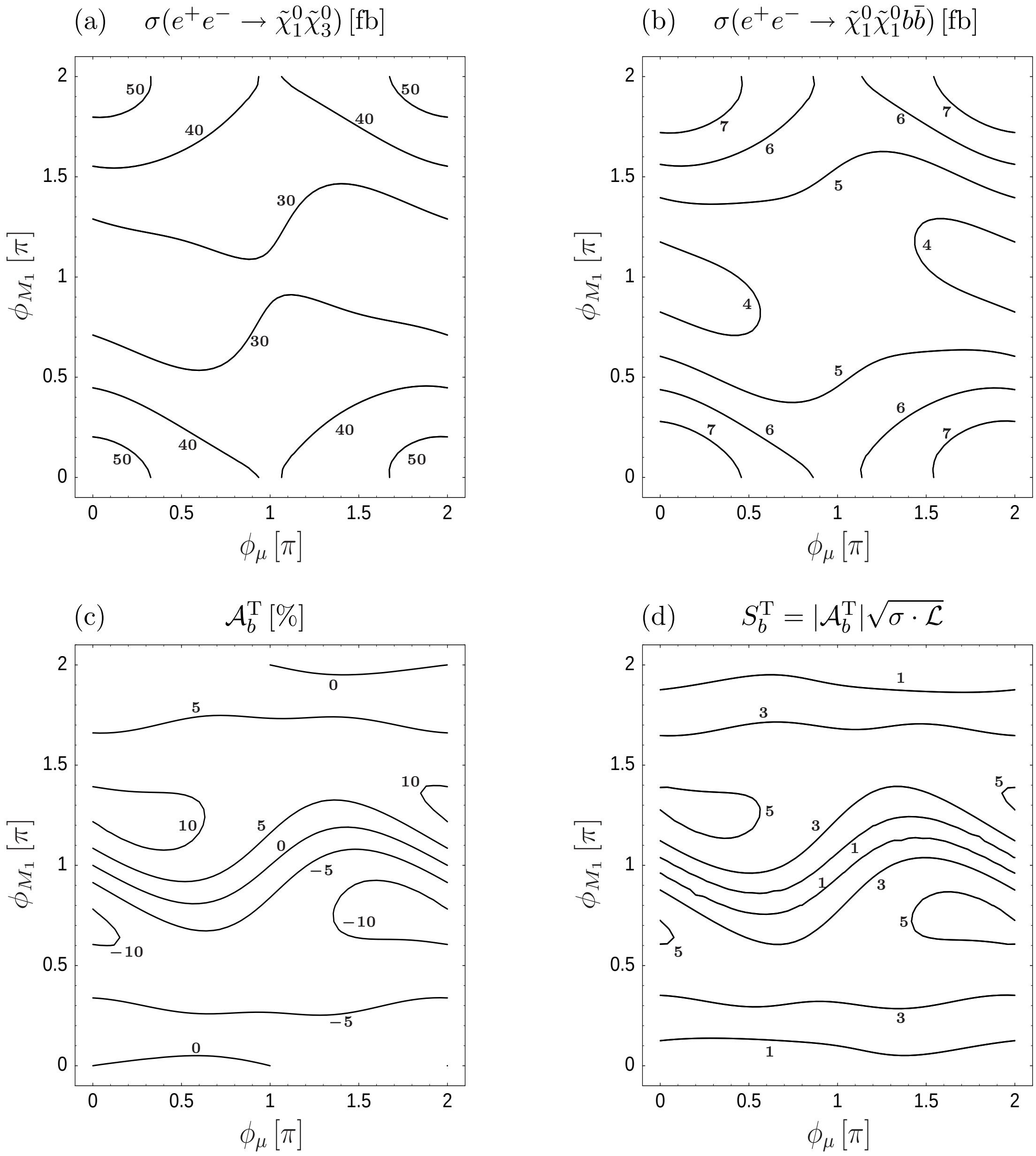

Figure 4: Neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ with transverse beam polarizations $\mathcal{P}_T(e^-) = 0.9$, $\mathcal{P}_T(e^+) = 0.6$, for Scenario B, given in table 1. Contour lines in the ϕ_μ–ϕ_{M_1} plane for (a) the production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$, (b) the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$, (c) the CP asymmetry $\mathcal{A}_b^{\mathrm{T}}$, and (d) the statistical significance S_b^{T}, for an integrated luminosity $\mathcal{L} = 500\,\mathrm{fb}^{-1}$.

In figure 8, we show the asymmetry $\mathcal{A}_b^{\mathrm{L}}$ and the cross section for longitudinally polarized beams $(\mathcal{P}_{e^-}^L, \mathcal{P}_{e^+}^L) = (-0.9, +0.6)$ in scenario C of table 1. The asymmetry $\mathcal{A}_b^{\mathrm{L}}$ can reach $\pm 10\%$, and the total cross section can reach $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b}) = 25\,\mathrm{fb}$. Again the strong ϕ_μ dependence of $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$ in the regions where $\phi_\mu \sim 0.4\pi, 1.6\pi$ is because the decay channels $\tilde{\chi}_2^0 \to \tilde{\ell}_R \ell$ are kinematical accessible beyond this threshold. The significance to measure $\mathcal{A}_b^{\mathrm{L}}$ for $(\phi_{M_1}, \phi_\mu) = (-\frac{\pi}{4}, \frac{\pi}{4})$ is $S_b^{\mathrm{L}} = 7$ ($\mathcal{L} = 500\,\mathrm{fb}^{-1}$). Once more we find that for longitudinally polarized beams, the asymmetry $\mathcal{A}_b^{\mathrm{L}}$ can be measured with

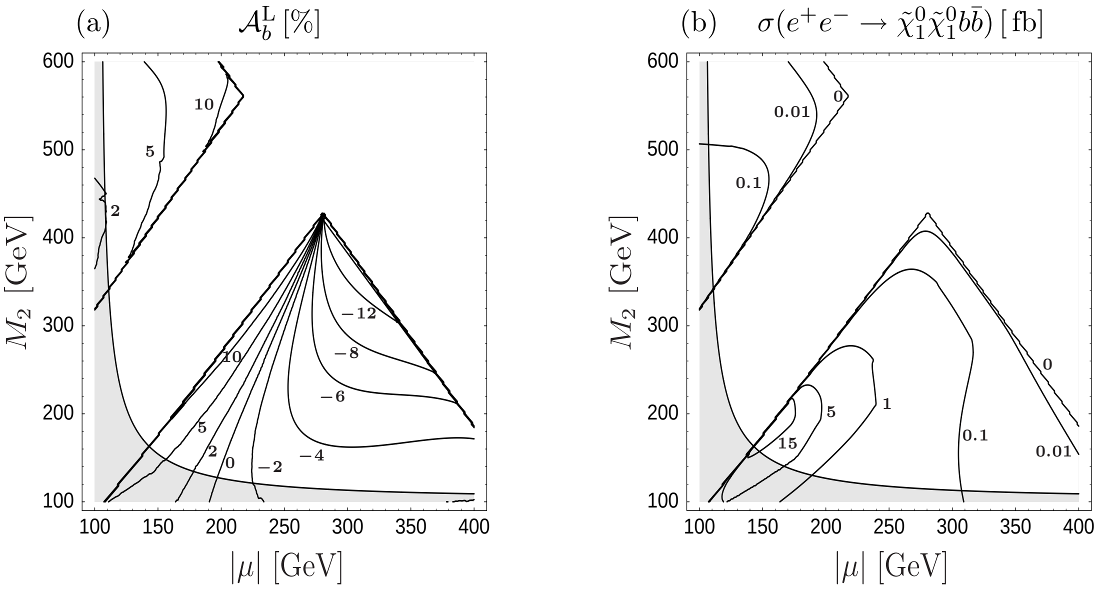

Figure 5: Neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ with longitudinal beam polarizations $\mathcal{P}_L(e^-) = 0.9$ and $\mathcal{P}_L(e^+) = -0.6$. Contour lines in the $|\mu|$–M_2 plane for (a) the CP asymmetry $\mathcal{A}_b^{\rm L}$, and (b) the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$, for Scenario A, given in table 1. The branching ratio $\mathrm{BR}(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0)$ is shown in figure 3(b).

higher statistics than the asymmetry $\mathcal{A}_b^{\rm T}$.

Finally, we have scanned the MSSM paramter space, and have found that in general the CP asymmetry $\mathcal{A}_b^{\rm T}$ is smaller for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production than for $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production. Note that in the parameter region where $M_2 > |\mu|$, for which the largest values of $\mathcal{A}_b^{\rm T}$ are obtained for $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production, the two-body decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$ is kinematically forbidden.

6. Summary and conclusions

We have studied the impact of the CP violating MSSM phases ϕ_{M_1} and ϕ_μ on neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and decay $\tilde{\chi}_i^0 \to \tilde{\chi}_n^0 Z^0$, $Z^0 \to f\bar{f}$. For transversely polarized beams, we have defined the CP observable $\mathcal{A}_f^{\rm T}$, which is an asymmetry in the azimuthal distribution of the final fermions, and is based on the triple product correlation eq. (4.1). The CP-asymmetry depends bilinearly on the beam polarizations $\mathcal{A}_f^{\rm T} \propto \mathcal{P}_T^-\mathcal{P}_T^+$, and receives CP sensitive contributions from spin-correlations in the neutralino production process. Due to the left-right structure of the Z^0 boson couplings to the final fermions, the asymmtry is largest $\mathcal{A}_b^{\rm T} = 6.3 \times \mathcal{A}_\ell^{\rm T}$ for the hadronic decay $Z^0 \to b\bar{b}$.

In a numerical analysis for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production, we have shown that the asymmetry can be as large as $\mathcal{A}_b^{\rm T} = 30\%$, for $\mathcal{P}_T(e^-) = 0.9$ and $\mathcal{P}_T(e^+) = 0.6$. The significance can be as large as $S_b^{\rm T} = 5$, with an integrated luminosity $\mathcal{L} = 500\,\mathrm{fb}^{-1}$. We have compared the cross sections and asymmetries with those which are accessible for longitudinally polarized beams. For $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production, we have found larger significances for the asymmetry $\mathcal{A}_f^{\rm L}$ with longitudinally polarized beams. Also for $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production the asymmetry $\mathcal{A}_f^{\rm L}$ is in general accessible with a larger statistical significance compared to the asymmetry $\mathcal{A}_f^{\rm T}$ based on transverse beam polarizations. This is mainly because longitudinal beam polarizations can

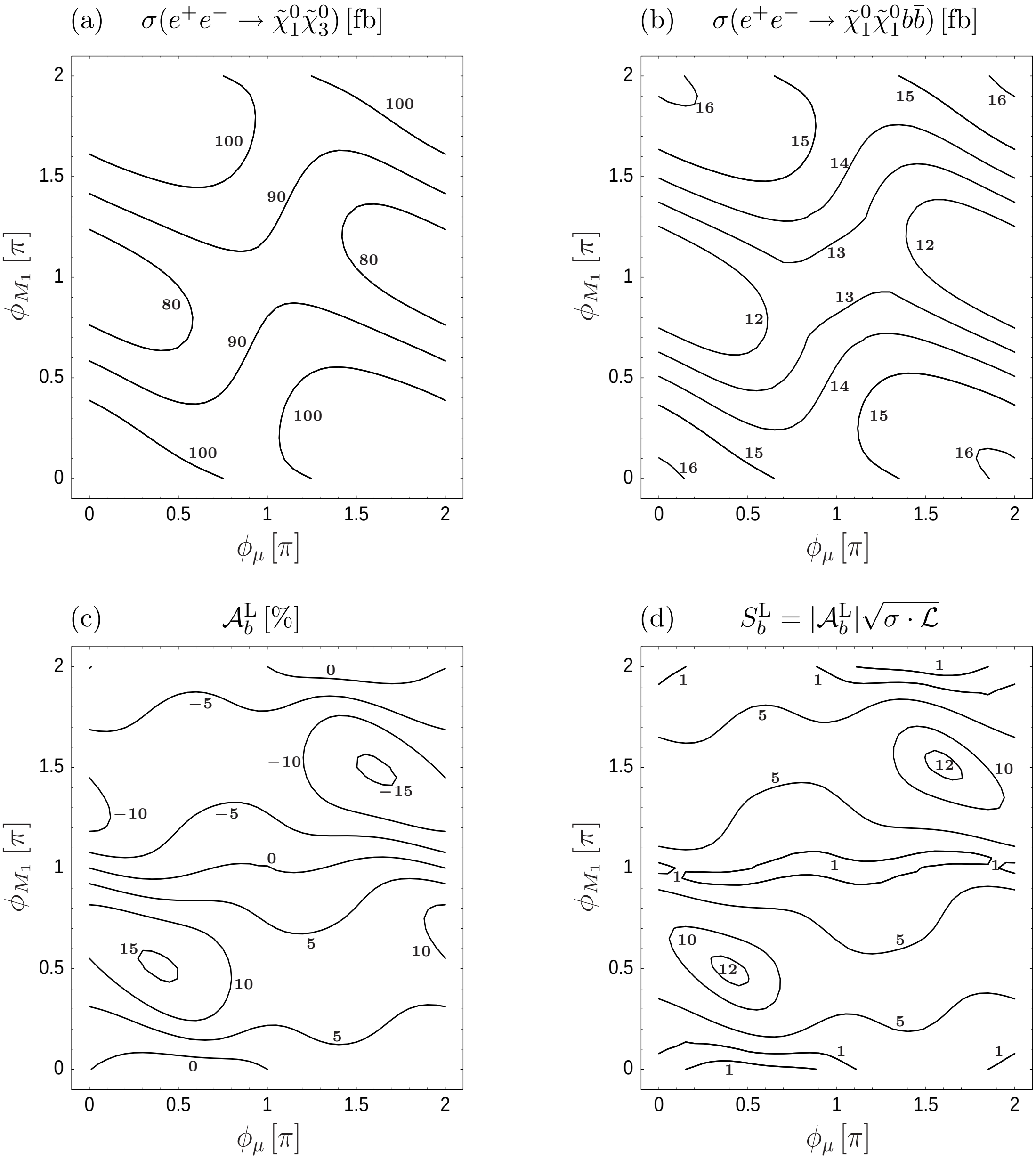

Figure 6: Neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and decay $\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ with longitudinal beam polarizations $\mathcal{P}_L(e^-) = 0.9$, $\mathcal{P}_L(e^+) = -0.6$, for Scenario B, given in table 1. Contour lines in the ϕ_μ–ϕ_{M_1} plane for (a) the production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$, (b) the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$, (c) the CP asymmetry $\mathcal{A}_b^{\mathrm{L}}$, and (d) the statistical significance S_b^{L}, for an integrated luminosity $\mathcal{L} = 500\,\mathrm{fb}^{-1}$.

greatly enhance the production cross section. There are, however, parameter regions, e.g. for $M_2 \gtrsim 300\,\mathrm{GeV}$ and $|\mu| \lesssim 200\,\mathrm{GeV}$, where the asymmetry $\mathcal{A}_f^{\mathrm{T}}$ in $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production can be up to about a factor five larger than the asymmetry $\mathcal{A}_f^{\mathrm{L}}$.

We conclude that for neutralino $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production and subsequent two-body decay into the Z^0 boson, longitudinally polarized beams give more statistics than transversely polarized beams ($S_f^{\mathrm{L}} > S_f^{\mathrm{T}}$) to study the corresponding CP asymmetry in the neutralino sector of the MSSM. However, for $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production, there are parameter regions where transversely

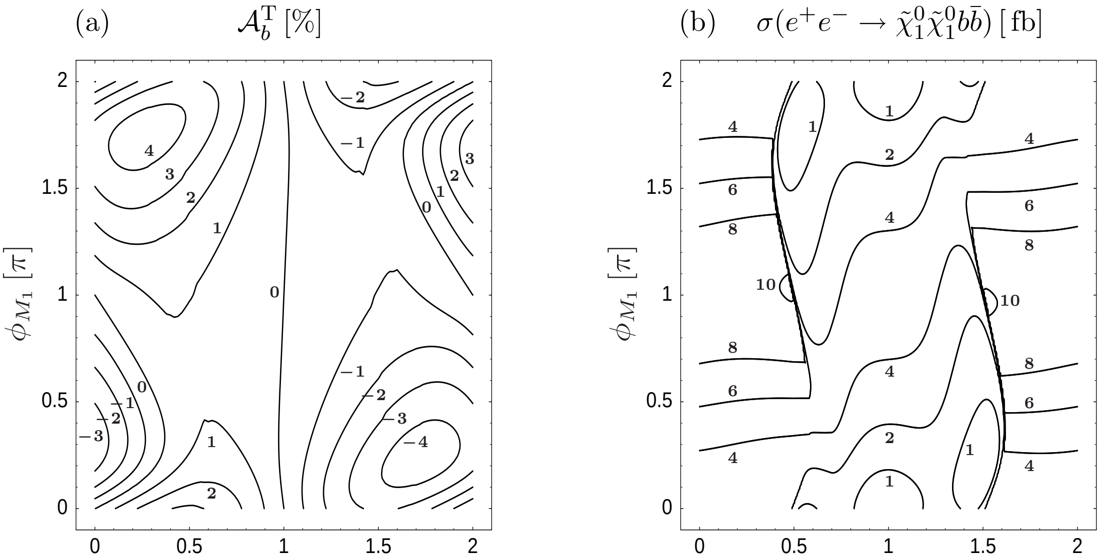

Figure 7: Neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ with transverse beam polarizations $\mathcal{P}_T(e^-) = 0.9$ and $\mathcal{P}_T(e^+) = 0.6$. Contour lines in the ϕ_μ–ϕ_{M_1} plane for (a) the CP asymmetry $\mathcal{A}_b^{\rm T}$, and (b) the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$, for Scenario C, given in table 1.

polarized beams are advantageous since $S_f^{\rm T} > S_f^{\rm L}$. We emphasize that the asymmetries for longitudinally und transversely polarized beams are independent of each other. They rely on different triple product correlations, and depend on different interference channels in the neutralino production process. Therefore both options of polarized beams should be considered to determine the phases ϕ_{M_1} and ϕ_μ by measurements of the asymmetries $\mathcal{A}_f^{\rm T}$ and $\mathcal{A}_f^{\rm L}$ at the ILC.

Acknowledgments

This work is supported by the 'Fonds zur Förderung der wissenschaftlichen Forschung' (FWF) of Austria, project. No. P18959-N16. The authors acknowledge support from EU under the MRTN-CT-2006-035505 network programme. This work was also supported by the SFB Transregio 33: The Dark Universe.

A. Momentum and polarization vectors

We introduce a coordinate frame in the laboratory system by choosing the z-axis along the momentum vector of the e^- beam, and x and y according to a right-handed coordinate system:

$$p_{e^-}^\mu = E_b(1,0,0,1) \quad \text{and} \quad p_{e^+}^\mu = E_b(1,0,0,-1)\,, \tag{A.1}$$

with the beam energy $E_b = \sqrt{s}/2$. Then the 4-vectors of the transverse polarization of the electron and positron beams are in the x–y plane

$$t_\pm = (0, \cos\phi_\pm, \sin\phi_\pm, 0)\,, \tag{A.2}$$

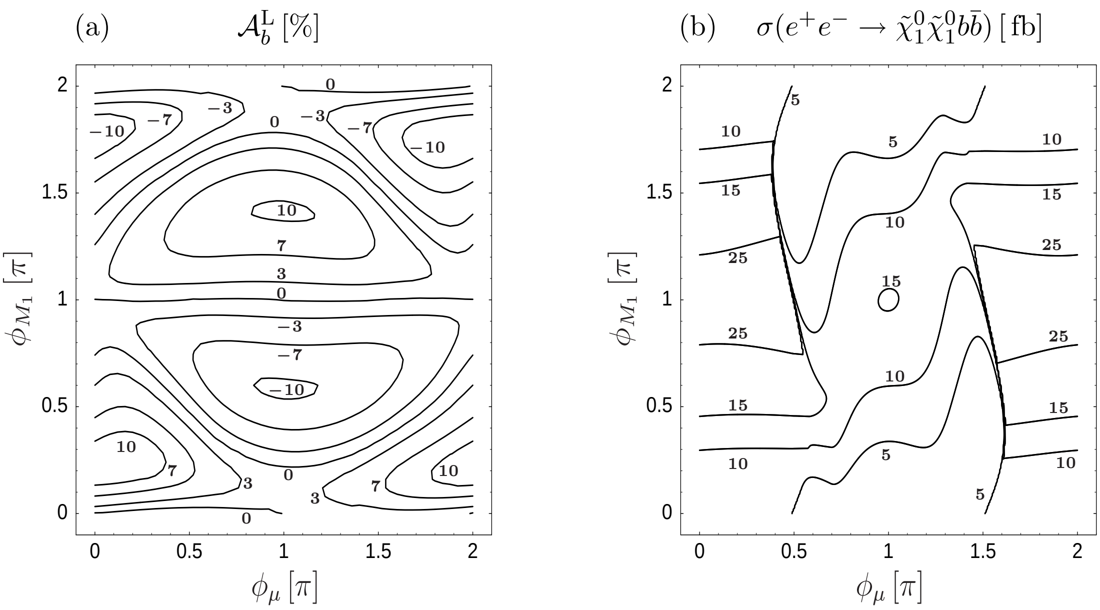

Figure 8: Neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$, $Z^0 \to b\bar{b}$ at $\sqrt{s} = 500\,\mathrm{GeV}$ with longitudinal beam polarizations $\mathcal{P}_L(e^-) = -0.9$ and $\mathcal{P}_L(e^+) = 0.6$. Contour lines in the ϕ_μ–ϕ_{M_1} plane for (a) the CP asymmetry $\mathcal{A}_b^{\mathrm{L}}$, and (b) the total cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 b\bar{b})$, for Scenario C, given in table 1.

where the azimuthal angles ϕ_+ and ϕ_- describe the orientation of the transverse beam polarizations, which are fixed at any value $\phi_+, \phi_- \in [0, 2\pi)$. The four-momenta of the neutralinos are

$$\begin{aligned} p^\mu_{\chi_j} &= q\left(\frac{E_{\chi_j}}{q}, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta\right), \\ p^\mu_{\chi_i} &= q\left(\frac{E_{\chi_i}}{q}, -\cos\phi\sin\theta, -\sin\phi\sin\theta, -\cos\theta\right), \end{aligned} \tag{A.3}$$

where θ is the scattering angle and ϕ the azimuthal angle of the production process. The energies and momenta of the neutralinos are

$$E_{\chi_{i(j)}} = \frac{s + m^2_{\chi_{i(j)}} - m^2_{\chi_{j(i)}}}{2\sqrt{s}}, \qquad q = \frac{\lambda^{\frac{1}{2}}(s, m^2_{\chi_i}, m^2_{\chi_j})}{2\sqrt{s}}, \tag{A.4}$$

with $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. The three spin-basis vectors $s^{b,\mu}_{\chi_j}$, $b = 1, 2, 3$, of neutralino $\tilde{\chi}_j^0$ are

$$\begin{aligned} s^{1,\mu}_{\chi_j} &= \left(0, \frac{\vec{s}^{\,2}_{\chi_j} \times \vec{s}^{\,3}_{\chi_j}}{|\vec{s}^{\,2}_{\chi_j} \times \vec{s}^{\,3}_{\chi_j}|}\right) = (0, -\cos\phi\cos\theta, -\sin\phi\cos\theta, \sin\theta), \\ s^{2,\mu}_{\chi_j} &= \left(0, \frac{\vec{p}_{\chi_j} \times \vec{p}_{e^-}}{|\vec{p}_{\chi_j} \times \vec{p}_{e^-}|}\right) = (0, \sin\phi, -\cos\phi, 0), \\ s^{3,\mu}_{\chi_j} &= \frac{1}{m_{\chi_j}}\left(q, \frac{E_{\chi_j}}{q}\vec{p}_{\chi_j}\right) = \frac{E_{\chi_j}}{m_{\chi_j}}\left(\frac{q}{E_{\chi_j}}, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta\right), \end{aligned} \tag{A.5}$$

with $s^b_{\chi_j} \cdot p_{\chi_j} = 0$ and $(s^a_{\chi_j} s^b_{\chi_j}) = -\delta_{ab}$, such that $\{\vec{s}^{\,1}_{\chi_j}, \vec{s}^{\,2}_{\chi_j}, \vec{s}^{\,3}_{\chi_j}\}$ build a right-handed system.

For the two-body decay $\tilde{\chi}_j^0 \to \tilde{\chi}_n^0 Z$, the decay angle $\theta_1 \angle(\vec{p}_{\chi_j}, \vec{p}_Z)$ is constrained by $\sin\theta_1^{\max} = q^0/q$ for $q > q^0$, where $q^0 = \lambda^{\frac{1}{2}}(m_{\chi_j}^2, m_Z^2, m_{\chi_n}^2)/(2m_Z)$ is the neutralino momentum if the Z^0 boson is produced at rest. In this case there are two solutions [43, 33, 29]

$$|\vec{p}_Z^{\pm}| = \frac{(m_{\chi_j}^2 + m_Z^2 - m_{\chi_n}^2) q \cos\theta_1 \pm E_{\chi_j}\sqrt{\lambda(m_{\chi_j}^2, m_Z^2, m_{\chi_n}^2) - 4q^2\, m_Z^2\, (1-\cos^2\theta_1)}}{2q^2(1-\cos^2\theta_1) + 2m_{\chi_j}^2}. \tag{A.6}$$

If $q^0 > q$, the decay angle θ_1 is not constrained and there is only the physical solution $|\vec{p}_Z^{\,+}|$. The momenta in the laboratory system are

$$p_Z^{\pm} = (E_Z^{\pm}, |\vec{p}_Z^{\,\pm}| \sin\theta_1 \cos\phi_1, |\vec{p}_Z^{\,\pm}| \sin\theta_1 \sin\phi_1, |\vec{p}_Z^{\,\pm}| \cos\theta_1)\,, \tag{A.7}$$

$$p_{\bar{f}} = (E_{\bar{f}}, |\vec{p}_{\bar{f}}| \sin\theta_2 \cos\phi_2, |\vec{p}_{\bar{f}}| \sin\theta_2 \sin\phi_2, |\vec{p}_{\bar{f}}| \cos\theta_2)\,, \tag{A.8}$$

$$E_{\bar{f}} = |\vec{p}_{\bar{f}}| = \frac{m_Z^2}{2(E_Z^{\pm} - |\vec{p}_Z^{\,\pm}| \cos\theta_D)}\,, \tag{A.9}$$

with $\theta_2 \angle(\vec{p}_{\chi_j}, \vec{p}_{\bar{f}})$ and the decay angle $\theta_D \angle(\vec{p}_Z, \vec{p}_{\bar{f}})$ given by

$$\cos\theta_D = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_2 - \phi_1)\ . \tag{A.10}$$

B. Phase space

The Lorentz invariant phase space element for the neutralino production (1.1) and the decay chain (1.2)–(1.3) can be decomposed into the two-body phase space elements [43, 33, 29]

$$\begin{aligned} d\text{Lips}(s, p_{\chi_i}, p_{\chi_n}, p_f, p_{\bar{f}}) &= \frac{1}{(2\pi)^2}\, d\text{Lips}(s, p_{\chi_i}, p_{\chi_j})\, ds_{\chi_j} \\ &\quad \times \sum_{\pm} d\text{Lips}(s_{\chi_j}, p_{\chi_n}, p_Z^{\pm})\, ds_Z\, d\text{Lips}(s_Z, p_f, p_{\bar{f}})\,, \end{aligned} \tag{B.1}$$

$$d\text{Lips}(s, p_{\chi_i}, p_{\chi_j}) = \frac{q}{4(2\pi)^2\sqrt{s}}\, d\Omega\,, \tag{B.2}$$

$$d\text{Lips}(s_{\chi_j}, p_{\chi_n}, p_Z^{\pm}) = \frac{1}{2(2\pi)^2}\, \frac{|\vec{p}_Z^{\,\pm}|^2}{2|E_Z^{\pm}\, q\cos\theta_1 - E_{\chi_j}\, |\vec{p}_Z^{\,\pm}||}\, d\Omega_1\,, \tag{B.3}$$

$$d\text{Lips}(s_Z, p_f, p_{\bar{f}}) = \frac{1}{2(2\pi)^2}\, \frac{|\vec{p}_{\bar{f}}|^2}{m_Z^2}\, d\Omega_2\,, \tag{B.4}$$

with $s_{\chi_j} = p_{\chi_j}^2$, $s_Z = p_Z^2$ and $d\Omega_i = \sin\theta_i\, d\theta_i\, d\phi_i$. We use the narrow width approximation for the propagators

$$\int |\Delta(\tilde{\chi}_j^0)|^2 ds_{\chi_j} = \frac{\pi}{m_{\chi_j}\Gamma_{\chi_j}}\,, \qquad \int |\Delta(Z^0)|^2 ds_Z = \frac{\pi}{m_Z\Gamma_Z}\ . \tag{B.5}$$

The approximation is justified for $\Gamma_{\chi_j}/m_{\chi_j} \ll 1$, which holds in our case with $\Gamma_{\chi_j} \lesssim \mathcal{O}(1\text{GeV})$. Note, however, that the naive $\mathcal{O}(\Gamma/m)$-expectation of the error can easily receive large off-shell corrections of an order of magnitude and more, in particular at threshold or due to interferences with other resonant or non-resonant processes. For a recent discussion of these issues, see [53, 54]

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